

## Exercise 11

Find the particular solution for each of the following initial value problems:

$$(1 + x^3)u' + 3x^2u = 1, \quad u(0) = 0$$

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### Solution

Observe that the left side can be written as  $[(1 + x^3)u]'$  by the product rule.

$$\frac{d}{dx}[(1 + x^3)u] = 1$$

Now integrate both sides with respect to  $x$ .

$$(1 + x^3)u = x + C$$

The general solution is thus

$$u(x) = \frac{x + C}{1 + x^3}.$$

Because an initial condition is given, this constant of integration can be determined.

$$u(0) = \frac{0 + C}{1 + 0} = C \quad \rightarrow \quad C = 0$$

Therefore,

$$u(x) = \frac{x}{1 + x^3}.$$